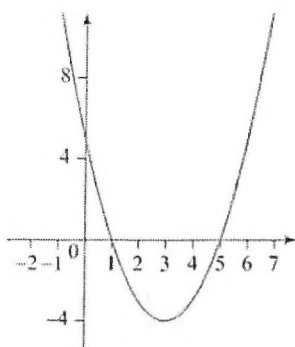
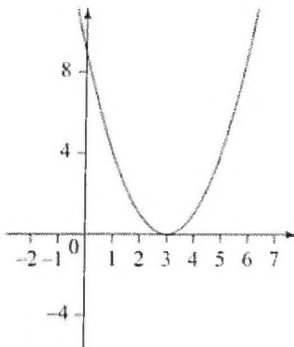


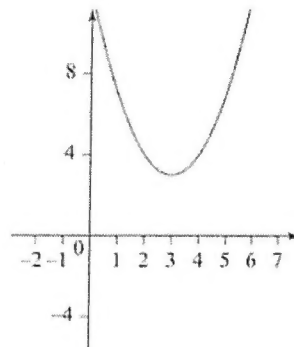
Quadratics - Discriminant Analysis



If $\Delta > 0$, the quadratic $y = ax^2 + bx + c$ has two distinct roots and the curve crosses the x -axis at two distinct points.



If $\Delta = 0$, the quadratic $y = ax^2 + bx + c$ has one repeated root and the x -axis is a tangent to the curve at this point.



If $\Delta < 0$, the quadratic $y = ax^2 + bx + c$ has no (real) roots and the curve does not cross the x -axis at any point.

1.

The quadratic equation $(k + 1)x^2 + 12x + (k - 4) = 0$ has real roots.

(a) Show that $k^2 - 3k - 40 \leq 0$. (3 marks)

(b) Hence find the possible values of k . (4 marks)

2.

(a) (i) Express $2x^2 - 20x + 53$ in the form $2(x - p)^2 + q$, where p and q are integers. (2 marks)

(ii) Use your result from part (a)(i) to explain why the equation $2x^2 - 20x + 53 = 0$ has no real roots. (2 marks)

(b) The quadratic equation $(2k - 1)x^2 + (k + 1)x + k = 0$ has real roots.

(i) Show that $7k^2 - 6k - 1 \leq 0$. (4 marks)

(ii) Hence find the possible values of k . (4 marks)

3.

The quadratic equation $x^2 + (m + 4)x + (4m + 1) = 0$, where m is a constant, has equal roots.

(a) Show that $m^2 - 8m + 12 = 0$. (3 marks)

(b) Hence find the possible values of m . (2 marks)

4.

Show that the quadratic equation

$$(k+1)x^2 + 2kx + k = 1$$

has two distinct real roots for all values of k , except for one value which must be stated.

5.

Find the range of values that the constant k can take so that

$$2x^2 + (k+2)x - k = 0$$

has two distinct real roots.

6.

Find the possible solutions of the quadratic equation

$$x^2 + (3-m)x + 5 = m^2,$$

where m is a constant, given that it has repeated roots.

7.

$$x^2 - 4ax + 2b + 1 = 0.$$

The above quadratic equation, where a and b are constants, has no real solutions.

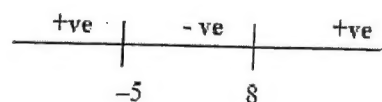
Show clearly that

$$b > \frac{1}{2}(2a+1)(2a-1).$$

Quadratics - Discriminant Analysis Solutions

1.

(a)	$b^2 - 4ac = 144 - 4(k+1)(k-4)$	M1		Clear attempt at $b^2 - 4ac$ Condone slip in one term of expression
	Real roots when $b^2 - 4ac \geq 0$	B1		Not just a statement, must involve k
	$36 - (k^2 - 3k - 4) \geq 0$			
	$\Rightarrow k^2 - 3k - 40 \leq 0$	A1	3	AG (watch signs carefully)
(b)	$(k-8)(k+5)$	M1		Factors attempt or formula
	Critical points 8 and -5	A1		
	Sketch or sign diagram correct, must have 8 and -5	M1		
	$-5 \leq k \leq 8$	A1	4	
	A0 for $-5 < k < 8$ or two separate inequalities unless word AND used			



2.

(a)(i)	$2(x-5)^2 + 3$	B1 B1	2	$p = 5$ $q = 3$
(ii)	Stating both $(x-5)^2 \geq 0$ and $3 > 0$ $\Rightarrow 2x^2 - 20x + 53 > 0$ or $2(x-5)^2 + 3 > 0$ $\Rightarrow 2x^2 - 20x + 53 = 0$ has no real roots	M1 A1cso	2	FT their p & q , but must have $q > 0$ Must have statement and correct p & q .
(b)(i)	$b^2 - 4ac = (k+1)^2 - 4k(2k-1)$ $= -7k^2 + 6k + 1$ real roots $\Rightarrow b^2 - 4ac \geq 0$ $-7k^2 + 6k + 1 \geq 0$ $\Rightarrow 7k^2 - 6k - 1 \leq 0$	M1 A1 B1✓ A1cso	4	Condone one slip (including x is one slip) Condone recovery from missing brackets Their discriminant ≥ 0 (in terms of k) Need not be simplified & may earn earlier AG (must see sign change)
(ii)	$(7k+1)(k-1)$ Critical values $k = 1, -\frac{1}{7}$	M1 A1		Correct factors or correct use of formula May score M1, A1 for correct critical values seen as part of incorrect final answer with or without working.

5.

$$x^2 + (k+2)x + k = 0$$

Two distinct roots $\Rightarrow b^2 - 4ac > 0$

$$\Rightarrow (k+2)^2 - 4 \times 2 \times k > 0$$

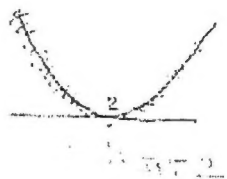
$$\Rightarrow (k+2)^2 - 8k > 0$$

$$\Rightarrow k^2 + 4k + 4 - 8k > 0$$

$$\Rightarrow k^2 - 4k + 4 > 0$$

$$\Rightarrow (k-2)^2 > 0$$

$$C.V = 2$$



$$\therefore x \in \mathbb{R}, x \neq 2$$

6.

$$x^2 + (3-m)x + 5 = m^2$$

$$x^2 + (3-m)x + (5-m^2) = 0$$

Repeated roots $\Rightarrow b^2 - 4ac = 0$

$$\Rightarrow (3-m)^2 - 4 \times 1 \times (5-m^2) = 0$$

$$\Rightarrow 9 - 6m + m^2 - 20 + 4m^2 = 0$$

$$\Rightarrow 5m^2 - 6m - 11 = 0$$

$$\Rightarrow (5m-11)(m+1) = 0$$

$$\Rightarrow m = \begin{cases} -1 \\ -\frac{11}{5} \end{cases}$$

• If $m = -1$

$$x^2 + (3-m)x + (5-m^2) = 0$$

$$x^2 + 4x + 4 = 0$$

$$(x+2)^2 = 0$$

$$x = -2$$

• If $m = -\frac{11}{5}$

$$x^2 + (3-m)x + (5-m^2) = 0$$

$$x^2 + \left(3 - \frac{11}{5}\right)x + \left(5 - \frac{121}{25}\right) = 0$$

$$x^2 + \frac{4}{5}x + \frac{4}{25} = 0$$

$$25x^2 + 20x + 4 = 0$$

$$(5x+2)^2 = 0$$

$$x = -\frac{2}{5}$$

7.

$$x^2 - 4ax + 2b + 1 = 0$$

(opposite sign to B^2 for DISCRIMINANT to avoid confusion)

$$\text{No real solutions} \Rightarrow B^2 - 4AC < 0$$

$$\Rightarrow (-4a)^2 - 4 \times 1 \times (2b + 1) < 0$$

$$\Rightarrow 16a^2 - 4(2b + 1) < 0$$

$$\Rightarrow 4a^2 - (2b + 1) < 0$$

$$\Rightarrow -(2b + 1) < -4a^2$$

$$\Rightarrow 2b + 1 > 4a^2$$

$$\Rightarrow 2b > 4a^2 - 1$$

$$\Rightarrow 2b > (2a - 1)(2a + 1)$$

$$\Rightarrow b > \frac{1}{2}(2a - 1)(2a + 1)$$